02 Information theory

02.03 Representation of non-numerical sets

- Texts
- Images
- Signals (Audio/Video)
- Redundancy and compression

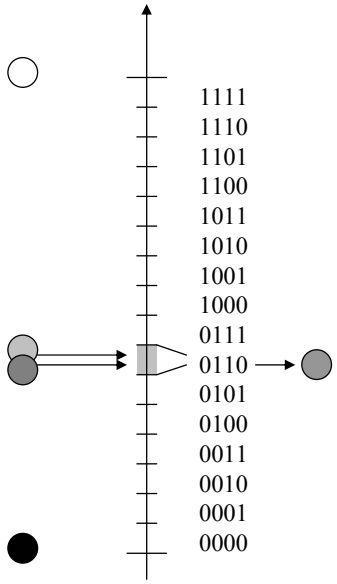
Text

1. A text is a sequence of characters
2. Each character is taken from a finite alphabete
3. Using a constant-size encoding for the characters, a text is encoded as a concatenation of character codes
4. ASCII: 7-bit encoding
5. Extended ASCII: 8-bit encoding

Images

1. An image is a matrix of points with assigned colors
2. An image contains infinite points and each point may take infinite colors
3. Both space and color discretization required
4. Discretized points are called *pixels*
5. Pixels are organized on a matrix
6. Using a constant size encoding for each pixel, an image is a concatenation of pixels, to be read in a given order

Color (gray) levels



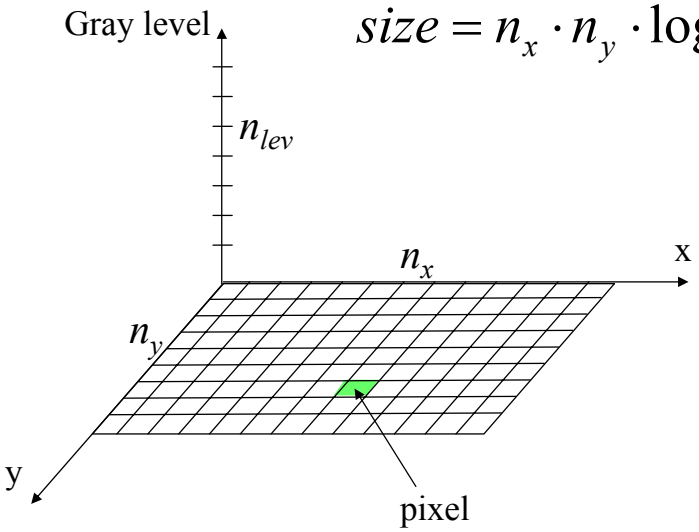
The encoding associates a unique code with an interval of gray levels

All gray levels within the interval are associated with the same code, thus loosing information

The original gray level cannot be exactly reconstructed from the code

Encoding associates each code with a unique gray level (representative of a class)

2D images



$$size = n_x \cdot n_y \cdot \log_2 n_{lev}$$

Example



100x100x8*bit*

100x100x1*bit*



50x50x8*bit*

50x50x1*bit*



10x10x8*bit*

10x10x1*bit*

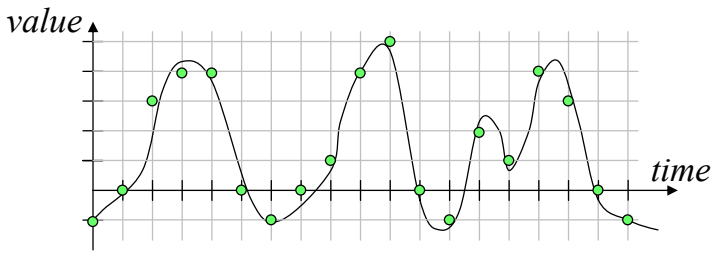

Analog and digital signals

- *Signal*: time-varying physical quantity
  - *Analog*: continuous-time, continuous-value
  - *Digital*: discrete-time, discrete-value
- The digital encoding of a continuous signal entails:
  - *Sampling* (i.e., time discretization)
  - *Quantization* (i.e., value discretization)

$$size = s_{rate} \cdot T \cdot s_{size}$$

Sampling rate — Duration — Sample size

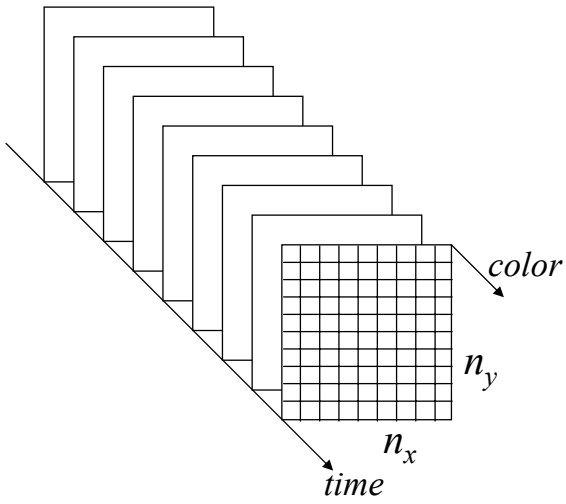
Audio: time series



$$size = s_{rate} \cdot T \cdot s_{size} = s_{rate} \cdot T \cdot \log_2 n_{lev}$$

Video

$$size = s_{rate} \cdot T \cdot s_{size} = s_{rate} \cdot T \cdot log_2 n_{col} \cdot n_x \cdot n_y$$



s_{rate} = frame rate
n_{col} = number of colors
$n_x n_y$ = frame size

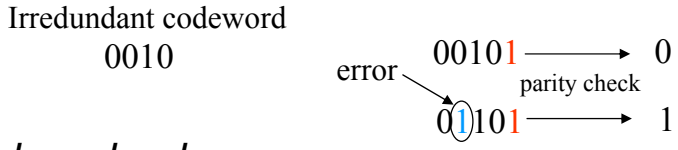
Redundancy

- *Redundant encoding*: encoding that makes use of more than the minimum number of digits required by an exact encoding $N > \lceil \log_S M \rceil$

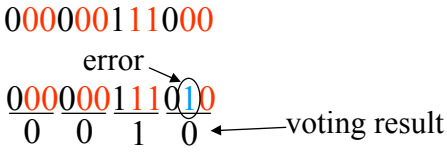
- Motivations for redundancy:
  - Providing more expressive/natural encoding/decoding rules
  - Reliability (error detection)
    Ex: *parity* encoding
  - Noise immunity / fault tolerance (error correction)
    Ex: *triplication*

Redundancy: examples

- *Parity encoding*:
  - A parity bit is used to guarantee that all codewords have an even number of 1's
  - Single errors are detected by means of a parity check



- *Triple redundancy*:
  - Each character is repeats 3 times
  - Single errors are corrected by means of a majority voting



Compression

- *Lossy* compression
  - Compression achieved at the cost of reducing the accuracy of the representation
  - The original representation cannot be restored
  - Always effective

- *Lossless* compression
  - Compression achieved by either removing redundancy or leveraging content-specific opportunities
  - The original representation can be restored
  - Not always effective