# PRECORSO DI MATEMATICA

## RICHIAMI TEORICI ED ESERCIZI SUI
## LOGARITMI

Siano $a$, $b \in \mathbb{R}$ con $a > 0$, $a \neq 1$ e $b > 0$.

$$x = \log_a b \quad \Longleftrightarrow \quad a^x = b\,.$$

$a$ si chiama **base** e $b$ **argomento** del logaritmo.

**Proprietà dei logaritmi:**

- $\log_a(b \cdot c) = \log_a b + \log_a c$

- $\log_a\left(\dfrac{b}{c}\right) = \log_a b - \log_a c$

- $\log_a(b^n) = n \log_a b\,, \quad n \in \mathbb{R}$

- $\log_a b = \dfrac{\log_r b}{\log_r c}\,, \quad r > 0\,, r \neq 1$

- $\log_a a = 1$

- $\log_a 1 = 0$

**Esercizio 1:** Calcolare il valore della seguente espressione

$$\log_2 \sqrt[4]{\frac{4\sqrt{2\sqrt[3]{4}}}{\sqrt[3]{2}}}\,.$$

*Svolgimento:* Usando le proprietà dei logaritmi si ha

$$\log_2 \sqrt[4]{\frac{4\sqrt{2\sqrt[3]{4}}}{\sqrt[3]{2}}} = \log_2 \left(\frac{4\sqrt{2\sqrt[3]{4}}}{\sqrt[3]{2}}\right)^{\frac{1}{4}}$$

$$= \frac{1}{4} \log_2 \frac{4\sqrt{2\sqrt[3]{4}}}{\sqrt[3]{2}}$$

$$= \frac{1}{4}\left[\log_2\left(4\sqrt{2\sqrt[3]{2^2}}\right) - \log_2 \sqrt[3]{2}\right]$$

$$= \frac{1}{4} \left[ \log_2 \left( 2^2 \sqrt{2 \cdot 2^{2/3}} \right) - \frac{1}{3} \log_2 2 \right]$$

$$= \frac{1}{4} \left[ \log_2 \left( 2^2 \cdot \left( 2 \cdot 2^{2/3} \right)^{\frac{1}{2}} \right) - \frac{1}{3} \right]$$

$$= \frac{1}{4} \left( \log_2 2^{17/6} - \frac{1}{3} \right)$$

$$= \frac{1}{4} \left( \frac{17}{6} \log_2 2 - \frac{1}{3} \right)$$

$$= \frac{1}{4} \left( \frac{17}{6} - \frac{1}{3} \right) = \frac{5}{8} .$$

**Esercizi:** Calcolare il valore delle seguenti espressioni

1. $\log_5 \sqrt{\dfrac{125}{5^7}}$

2. $\log_a \dfrac{a \sqrt[3]{a}}{\sqrt{a \sqrt{a^3}}}$

3. $\log_2 \left( \sqrt[4]{2} \cdot \sqrt[3]{4} \right)$

4. $\log_3 \dfrac{81 \sqrt[5]{27}}{\sqrt{3}}$

5. $\log_2 \dfrac{16 \sqrt[3]{2}}{\sqrt{\sqrt[4]{8}}}$

6. $\log_2 \left( 4 \sqrt[3]{\dfrac{8\sqrt{4\sqrt{8}}}{\sqrt{32\sqrt[3]{2}}}} \right)$

7. $\log_2 \left( 16 \cdot 2^9 \sqrt{2} \right)$

8. $\log_a \left( \dfrac{a^5 \sqrt{a}}{\sqrt[3]{a^2}} \right)$

9. $\log_9 \left( 3\sqrt{3} \dfrac{\sqrt[4]{27}}{\sqrt[5]{81}} \right)$

10. $\log_4 \left( \sqrt[3]{4} \cdot 4^5 \right)$

11. $\log_8 \left( 2 \sqrt[3]{\dfrac{\sqrt{32}}{2\sqrt[3]{2}}} \right)$

12. $\log_{\frac{1}{3}} \sqrt{\dfrac{\sqrt{\sqrt{27\sqrt{3}}}}{9\sqrt{3}}}$

13. $\log_4 \sqrt[5]{\dfrac{4\sqrt{2\sqrt{2}}}{\sqrt[3]{4\sqrt{2}}}}$

14. $\log_3 \left( 9\sqrt{3} \right)$

15. $\log_a \left( a\sqrt{a\sqrt{a}} \right)$

**Esercizio 2:** Semplificare la seguente espressione

$$\log \frac{\sqrt[4]{a^2 b^3 \sqrt[3]{ab}}}{\sqrt{ab\sqrt{a}}} \,.$$

*Svolgimento:* Usando le proprietà dei logaritmi si ha

$$\log \frac{\sqrt[4]{a^2 b^3 \sqrt[3]{ab}}}{\sqrt{ab\sqrt{a}}} = \log \frac{\sqrt[4]{a^2 b^3 \, (ab)^{1/3}}}{\sqrt{ab \cdot a^{1/2}}}$$

$$= \log \frac{\sqrt[4]{a^{2+1/3} \cdot b^{3+1/3}}}{\sqrt{a^{1+1/2} \cdot b}}$$

$$= \log \frac{\left( a^{7/3} \cdot b^{10/3} \right)^{\frac{1}{4}}}{\left( a^{3/2} \cdot b \right)^{\frac{1}{2}}}$$

$$= \log \left( a^{7/12 - 3/4} \cdot b^{5/6 - 1/2} \right)$$

$$= \log \left( a^{-1/6} \cdot b^{1/3} \right)$$

$$= \log a^{-1/6} + \log b^{1/3}$$

$$= -\frac{1}{6} \log a + \frac{1}{3} \log b \,.$$

**Esercizi:** Semplificare le seguenti espressioni

1. $\log \left( x^2 \sqrt[4]{\dfrac{xy}{\sqrt{x}} : \sqrt{\dfrac{x}{y}}} \right)$

2. $\log \sqrt[4]{\dfrac{ab^2 c \sqrt[3]{a^2 b}}{\sqrt{abc}} : \sqrt{a\sqrt{b\sqrt{c}}}}$

3. $\log \dfrac{ab^2 \sqrt{b}}{\sqrt{a\sqrt{ab}}}$

4. $\log \left[ \sqrt{\dfrac{a\,(a^2-1)\,\sqrt{a\sqrt{a}}}{\sqrt{a+1}}} : \sqrt{a\,(a-1)} \right]$

5. $\log \left( \sqrt{3}x\,(x+y) \right)$

6. $\log \left[ 3\,(x+1)^3 \, \sqrt[4]{\dfrac{\sqrt{x^2-1}}{3x+3}} : \sqrt{3x-3} \right]$

7. $\log_8 \left( 4\sqrt[3]{\dfrac{8\sqrt[4]{2}}{2}} \right) - 2\log_{\frac{1}{3}} \left( \dfrac{1}{27} \sqrt[3]{9\sqrt[4]{\dfrac{1}{81}} : \sqrt{3}} \right) + \log_9 \left( 3\sqrt[3]{\dfrac{1}{3}} \right)$

8. $\log \sqrt[4]{a^2 \dfrac{3\sqrt[3]{a}}{a+3}}$

9. $\log \left[ \sqrt{(a-1)\sqrt{(a-1)\sqrt{a^2-1}}} : \sqrt[3]{(a^2-1)^2} \right]$

10. $\log \left( \dfrac{3\sqrt{a}}{\sqrt{ac}} \sqrt{\dfrac{a-2}{2ac}} \right)$

**Esercizio 3:** Ridurre ad un unico logaritmo la seguente espressione

$$2 \left( \log 2 - \frac{1}{2}\log 3 \right) + \frac{1}{2}\left( \log 3 - 3\log 2 \right).$$

*Svolgimento:* Usando le proprietà dei logaritmi si ha

$$
\begin{aligned}
2 \left( \log 2 - \frac{1}{2}\log 3 \right) + \frac{1}{2}\left( \log 3 - 3\log 2 \right) \;&= 2\left( \log 2 - \log 3^{1/2} \right) + \frac{1}{2}\left( \log 3 - \log 2^3 \right) \\[2mm]
&= 2\log \frac{2}{\sqrt{3}} + \frac{1}{2}\log \frac{3}{8} \\[2mm]
&= \log \left( \frac{2}{\sqrt{3}} \right)^2 + \log \left( \frac{3}{8} \right)^{1/2} \\[2mm]
&= \log \frac{4}{3} + \log \sqrt{\frac{3}{8}} \\[2mm]
&= \log \left( \frac{4}{3}\sqrt{\frac{3}{8}} \right) = \log \sqrt{\frac{2}{3}}\;.
\end{aligned}
$$

**Esercizi:** Ridurre ad un unico logaritmo le seguenti espressioni

1. $\log a - 3\log b + 2\left(\log a - \log b\right)$

2. $x\log 4 - (x-1)\log 2 + 3\left(x\log 2 - \log 3\right) + (x+1)\log 3$

3. $\dfrac{1}{3}\log m - \dfrac{1}{2}\log n - \dfrac{1}{2}\left(\log m + \dfrac{2}{5}\log n\right)$

4. $\dfrac{2}{3}\left[\log\left(a-b\right) - \dfrac{2}{3}\left(\log a + \log b\right) + 3\log a\right]$

5. $\dfrac{3}{4}\log x - \dfrac{1}{2}\left[\log x - 2\left(\log x + \dfrac{1}{3}\log x\right)\right]$

6. $\dfrac{1}{4}\left[\dfrac{1}{3}\left(\dfrac{1}{2}\log x + \log y\right) + 3\log x - \log y\right]$

7. $2\log b + \dfrac{1}{2}\left[\log a - \dfrac{1}{2}\left(\log a - \log b\right)\right]$

8. $\log a + \dfrac{1}{3}\left\{2\log a + \dfrac{1}{2}\left[2\log a - \log\left(2-a\right)\right]\right\}$

9. $\log 6 - \dfrac{2}{3}\log 27 + \log 3 - 2\log 2$

10. $\dfrac{1}{2}\log\left(1-x\right) - \dfrac{1}{2}\log\left(1-x^2\right) + 2\log x$